The Influence of location of balls and ball diameter difference in rolling bearings on the nonrepetitive runout (NRRO) of retainer revolution

Shoji Noguchi a,1, Kentaro Hiruma a,1, Hiroyuki Kawa b,2, Tohru Kanada a,∗

a Department of Mechanical Engineering, Tokyo University of Science, 2641 Yamazaki, Noda-city, Chiba 278-8510, Japan
b Department of Mechanical Engineering, Kanto Gakuin University, 1-50-1 Mutsuura-higashi, Kamagaya, Yokohama-city, Kanagawa 238-8501, Japan

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Abstract

The nonrepetitive runout (NRRO) value of retainer revolution is caused mainly by the diameter differences of balls mounted in a bearing. Additionally, when more than one ball has diameter difference, the NRRO value of retainer revolution is believed to vary with the location of balls. In this study, the authors theoretically analyzed the NRRO value of retainer revolution considering the diameter differences and location of balls mounted in a bearing. Consequently, it is clarified that the mean value of the retainer revolution component calculated in all locations of balls decreased with increasing number of balls in a rolling bearing.

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1. Introduction

The requirement of high accuracy in rotation is increasing for bearings in recent information equipment and machine tools. In particular, in the ball bearings used in a spindle motor for a magnetic disc drive unit, the requirement for reduction in the nonrepetitive runout (NRRO) component in the radial direction is increasing. Nowadays, a NRRO value below 0.05 μm is necessary to satisfy the above requirement. In order to reduce the NRRO value, minimizing form errors and dimensional errors of inner and outer raceways is important. However, improvement of working accuracy increases the manufacturing cost. Therefore, it is essential that we should investigate the incidence of contributing factors in order to improve the rotational errors efficiently.

The authors have clarified the quantitative relationship between the NRRO and geometrical errors (forms and dimensions) of various parts of ball bearings theoretically and experimentally [1,2]. In the actual experimental observation, the frequency of retainer (cage) revolution (fc component for short) is the largest factor. Therefore, NRRO value means fc component in this paper. The cause-and-effect relationship between vibration frequency of a rolling bearing and its regional geometrical errors leading to the vibration has already been explained [3]. The clarification will aid in the maintenance of machines. The main reason for the resulting fc component is diametrical differences (referred to as “mutual diameter differences of balls”) of balls mounted in a ball bearing. When a ball bearing is constructed, if the ball diameters are measured one by one and the balls with the same diameter are lined up, the fc component will become smaller. But, this raises a cost issue. In the present circumstances, the mutual diameter differences of balls are controlled in units of production lots. For example, the production lot consists of several hundreds of thousand of balls for 2 mm diameter. Moreover, it is clarified theoretically that the fc component is influenced by the location of balls. Balls of a predetermined number taken from several hundreds of thousand of balls in a production lot are fitted into a ball bearing. Therefore, statistical analysis is important in order to clarify the influence of the location of balls.

This research deals with the following two results.

(1) Statistical analysis for determining the influence of mutual diameter differences of balls with the location of balls in the fc component.

(2) Experimental investigation for determining the influence of the location of balls in the fc component.
The radial NRRO is a problem to be investigated from the viewpoint of ball bearings used in magnetic disc devices. In this research, on the assumption that the ball bearing can be treated as a two-dimensional model, the radial behavior of the center position is analyzed considering the dynamical balance of the Herzian contacting stress between the inner and outer raceways and balls.

Fig. 1 shows a NRRO analytical model of the ball bearing. The X- and Y-axes run across the center, O', of the surface of the outer raceway, and x and y represent the coordinates from O', which is the center of the surface of the inner raceway. Since the geometrical errors of the surfaces of the raceways and the balls are small, it is assumed that the balls never slide, but roll perfectly, and have elastic contact with the surfaces of the inner and outer raceways. When the angle of the running inner raceway is \( \omega_{b} \), the angles of balls’ positions and the angle of balls’ rotation are given by the following equations.

Position of first ball : \( \Omega_{0} = \omega_{0} + \frac{1 - D/d}{2} \) (1)

Position of kth ball : \( \theta_{k} = \Omega_{0} + 2\pi \frac{k - 1}{2} \) (2)

Rotation angle of ball : \( \phi_{b} = \omega_{0} \frac{d_{b}}{D} = d_{b}/D \) (3)

Here, \( d_{b} \) is the diameter of the balls, and \( D \) is the diameter of the pitch circle of located balls. Letting \( d_{b}(\omega_{b}) \) be the diameter of the k-th ball at the contact point, and \( r(\theta_{b}) \) and \( R(\theta_{b}) \), be the radius of the outer and inner rings, respectively, the elastic displacement, \( \delta_{k} \), of the k-th ball when the axis of the shaft has shifted to the point \( (x, y) \) is given by

\[
\delta_{k} = r(\theta_{b}) + d_{b}(\omega_{b}) - R(\theta_{b}) - x \cos \theta_{b} + y \sin \theta_{b}
\]

Since it is assumed that the deformations of the outer and inner raceway surfaces are identical, \( r(\theta_{b}) \), \( R(\theta_{b}) \) and \( d_{b}(\omega_{b}) \) are given by the following equations as expressed in terms of the Fourier series with the addition of the basic radius and the circumferential geometrical errors.

\[
r(\theta_{b}) = r_{0} + \sum_{k=1}^{N} a_{k} \cos(n\theta_{b} + \phi_{n})
\]

\[
R(\theta_{b}) = R_{0} + \sum_{k=1}^{N} a_{k} \cos(n\theta_{b} + \phi_{k})
\]

\[
d_{b}(\omega_{b}) = 2r_{0} + \sum_{k=1}^{N} a_{k} \cos(m\omega_{b} + \phi_{k})
\]

where, \( (r_{0}, a_{k}, \phi_{n}, \omega_{b}) \) and \( (R_{0}, a_{k}, \phi_{k}, \omega_{b}) \) are the coefficients in the Fourier series expansion. From the Herzian theory of elastic contact, the force \( P_{k} \) working on the balls and the outer and inner raceways for elastic displacement, \( \delta_{k} \), is given by

\[
P_{k} = C_{k}\delta_{k}^{5}
\]

where \( C \) is a constant. Since the forces working on the balls and on the outer and inner raceways vary with the positions of the balls, the shaft axis shifts to a point where the resultant of the forces as vectors is zero. On account of this balancing relationship between the internal forces, the coordinates \( (x, y) \) of the axis can be obtained simultaneously by solving the following two equilibrium equations.

\[
\sum_{k=1}^{N} C_{k}\delta_{k}^{4} \cos \theta_{b} = 0 \quad (5)
\]

\[
\sum_{k=1}^{N} C_{k}\delta_{k}^{4} \sin \theta_{b} = 0 \quad (6)
\]

Since Eq. (5) and (6) are nonlinear with respect to changing angle of rotation of the inner ring, the motion of the shaft axis which corresponds to the changing angle of rotation can be
Table 1

<table>
<thead>
<tr>
<th>Conditions of calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.C.D. of located balls</td>
</tr>
<tr>
<td>Ball diameter (normal)</td>
</tr>
<tr>
<td>Initial elastic displacement</td>
</tr>
<tr>
<td>Number of data per one round</td>
</tr>
<tr>
<td>Number of rounds for NRRO calculation</td>
</tr>
<tr>
<td>Criteria of Newton–Raphson method</td>
</tr>
</tbody>
</table>

found by means of the Newton–Raphson method. In order to evaluate the calculated NRRO, Lissajous’ plot is applied. As shown in Fig. 2, all the round trip behaviors of the axis are plotted one after another, and the radial widths of the lines are defined as NRRO values.

3. Calculated results

3.1. Assumption

Table 1 shows the parameters used in the calculation. Required specifications of the bearing are pitch circle diameter (PCD) of the located balls, ball diameter and initial elastic displacement. All the balls contact with inner and outer raceways. The geometrical errors are configured to not go beyond the initial elastic displacement. Though the \( f_c \) component is related to the anis-location, that is, angle error from the predefined position [4], the main reason for \( f_c \) component is the mutual diameter differences of balls. Therefore, it is assumed that the inner and outer raceways are geometrically real circles and the balls are geometrically real sphere which have no form errors.

3.2. When one ball has a diameter difference

Fig. 3 shows the calculated \( f_c \) component in the case when one ball has a larger diameter by 0.05 μm than the reference diameter (1.2 mm). Though the \( f_c \) component decreases with increasing number of balls mounted in a bearing, the decrease ratio is larger when the number of balls is small, and is smaller when the number of balls is larger. Furthermore, the \( f_c \) component converges to about one fifth of the given diameter difference with a larger number of balls.

3.3. When multiple balls have the same diameter difference

Fig. 4 shows the calculated \( f_c \) component when multiple balls have the same diameter difference in the case that the number of balls is eight. In this case, the \( f_c \) component changes significantly according to the location of balls, and the following tendency can be recognized.

(1) The \( f_c \) component increases in proportion to the given diameter difference.

(2) If the balls having diameter differences are located serially, the \( f_c \) component becomes larger than that in the case of when one ball has a diameter difference.

(3) If the balls having diameter differences are located separately, the \( f_c \) component becomes smaller than that in the case of when one ball has a diameter difference. Then, in the case of the location of balls at equi-angles such as 180 or 90°, the \( f_c \) component becomes zero.

Thus, the \( f_c \) component is caused by nonuniformity of contacting stress between the raceways and balls. In the case that one ball has a diameter difference, the number of contacting points becomes larger when the number of balls increases. Therefore, the \( f_c \) component decreases, because the circumferential stress nonuniformity decreases. In the case that mul-
tiple balls have a diameter difference, if the balls are located serially, the \( f_c \) component increases, because the contacting stress nonuniformity is enlarged. On the other hand, in the case that the balls are located separately, the contacting stress nonuniformity is decreased. If the balls are located symmetrically about a point, the circumferential stress distribution assumes the shape of point symmetry. Then, the \( f_c \) component becomes zero, because the equable force acts from up and down, and side to side, so that the inner raceway does not travel radially. In the actual ball bearing, the contacting stress is reduced by the axial displacement of the inner raceway for that there is some contact angle.

3.4. When all the balls have different diameters

All the balls in one ball bearing have different diameter values because the predefined number (generally seven to dozen) of randomized balls, from one production lot of several hundreds of thousand of balls at most, are mounted into one ball bearing. Therefore, in order to investigate the influence of the location of balls for the \( f_c \) component, statistically expected value of the \( f_c \) component should be calculated.

3.4.1. Number of location patterns of balls acting on the \( f_c \) component

Let us consider the total number of combination patterns for location of balls acting on the \( f_c \) component. In the case of a ball bearing, the following can be stated.

1. The first ball and the last one are neighbors.
2. Rotational error values in clockwise and counterclockwise rotations are the same.

Therefore, the total number of patterns is one half of such a beadroll permutation. Let the number of balls be \( Z \), then the total number, \( N \), of location patterns of balls is expressed as follows.

\[
N = \frac{(Z!)^2 - 1}{2} \quad (7)
\]

Table 2 shows the data of the number of balls, number of patterns and elapsed time of calculation. The CPU of the computer used for the calculation is a Pentium 4 (1.8 GHz). For \( Z = 3 \), \( N = 1 \). In contrast, for \( Z = 10 \), \( N = 18,1440 \).

<table>
<thead>
<tr>
<th>Number of balls, ( Z )</th>
<th>Number of patterns</th>
<th>Elapsed time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>360</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>2520</td>
<td>36</td>
</tr>
<tr>
<td>9</td>
<td>20,140</td>
<td>243</td>
</tr>
<tr>
<td>10</td>
<td>181,440</td>
<td>2525</td>
</tr>
<tr>
<td>11</td>
<td>1,814,400</td>
<td>25,208</td>
</tr>
<tr>
<td>12</td>
<td>19,958,400</td>
<td>277,210</td>
</tr>
</tbody>
</table>

There is a greater increase in \( N \) for a large number of \( Z \). In addition, the elapsed time of the \( f_c \) component for 72 patterns is approximately one second.

The generation method for the location patterns of balls is shown in Fig. 5. Considering a polygon whose vertices are the centers of the balls, we can create a new location pattern of balls nonautologically inserting a new ball into the edge connecting the existing two balls. In the case of \( Z = 3 \), there is only one pattern. Therefore, if the fourth ball is inserted into one of the three edges (\( Z = 3 \), triangle), three location patterns of balls can be generated.

The process to obtain the expected value of the \( f_c \) component is as follows.

1. Let the balls be numbered virtually.
2. Let the balls have adequate diameter differences. This means that all the balls have different diameters.
3. Calculate the \( f_c \) component for all the combinations of location patterns of balls.
4. Divide the sum total of the \( f_c \) components by the total number of patterns.

3.4.2. When the given diameter differences are equally separated

Let the minimum–maximum width of balls diameters be \( 0.05 \mu \text{m} \). Then, assuming that each ball diameter has
a normal distribution (Gaussian). Fig. 6 shows the calculated expected value of the $f_c$ components. Here, the normal distribution of diameter is treated as follows. The minimum–maximum width, 0.05 μm, equals $6\sigma$ ($\sigma$, standard deviation) = ± 3σ, in other words ± 0.025 μm. The interval of diameter differences is 0.05 μm/Z (Z, number of balls).

In comparison with Fig. 3, which shows the result in the case that only one ball has a larger diameter by 0.05 μm, the expected value decreases. Moreover, the reduction ratio of the $f_c$ component also decreases with increasing number of balls.

When the $f_c$ component is maximized and minimized for $Z = 10$, the location pattern of balls is shown in Fig. 7. When the $f_c$ component is maximized, the larger balls are lined up serially and the smaller balls are lined up at the opposite faces. Furthermore, when the $f_c$ component is minimized, a large ball and a small ball are located neighboring each other in order to negate the diameter differences.

### 3.4.3. Considering mutual diameter differences of balls in a production lot

Balls numbering several hundreds of thousand for rolling bearing are manufactured in a production lot. The required accuracy for each grade and each lot is standardized as in Table 3 [5]. Here, variation of diameter means the maximum (peak) to minimum (valley) value of diameters in one ball. Furthermore, diameter difference per one lot means the maximum diameter value to the minimum diameter value in all the balls of one lot. From the above-mentioned results, it is clear that the mutual diameter differences of balls should be reduced in order to lower the $f_c$ component. However, it is difficult in terms of technique and cost to let the mutual diameter differences of balls be lower than 0.1 μm. So, first, the authors determined the number of balls. Second, the influence of the mutual diameter differences of balls in a production lot is calculated to investigate the effect of reduction of $f_c$ component.

Measurement of diameters for all the balls in a production lot is impossible. Thus, it is appropriate that the control of mutual diameter differences of balls should be performed by an estimation of population standard deviation from the random samples’ standard deviation. Let the standard deviation of population (production lot) be $\sigma_1$, and that of the number of random samples’ $\sigma_2$. The relationship between $\sigma_1$ and $\sigma_2$ is expressed as follows.

$$\sigma_1 = \sigma_2 \sqrt{\frac{Z}{Z - 1}}$$

<table>
<thead>
<tr>
<th>Grade</th>
<th>Variation of diameter</th>
<th>Sphericity (Ra)</th>
<th>Diameter difference per one lot (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.08</td>
<td>0.08</td>
<td>0.012</td>
</tr>
<tr>
<td>5</td>
<td>0.13</td>
<td>0.13</td>
<td>0.02</td>
</tr>
<tr>
<td>10</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>16</td>
<td>0.4</td>
<td>0.4</td>
<td>0.032</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>
In the actual calculation, the mutual diameter differences of balls in a production lot (namely the value of $6\sigma_1$) is determined in advance. Then, in order to let the standard deviation, $\sigma_2$, satisfy Eq. (8), the following conditions are configured [6,7].

1. Distribution shape of the ball diameter is symmetric around the reference diameter.
2. Each ball diameter is set such that the interval of diameter differences is equal.

For example, if the mutual diameter differences of balls, $6\sigma_1 = 0.06 \mu m\ (\sigma_1 = 0.01 \mu m)$ and $Z = 10$, then $\sigma_2 = 0.01 \times (9/10)^{1/2} = 0.0095 \mu m$. Each ball diameter of $Z$ is defined to satisfy the above-mentioned conditions (1) and (2). Fig. 8 shows the results of the expected value of the $f_c$ component for $Z = 7, 8$ and $10$. JIS B 1501 (ISO 3290) describes that grade 3 has the highest accuracy, that is, mutual diameter differences of balls are up to $0.08 \mu m$. However, further accuracy is realized in the balls of rolling bearings used in the hard disc drives and spindle motors. Therefore, the mutual diameter differences of balls down to $0.01 \mu m$ are applied in the calculation of the $f_c$ component. Incidentally, the roundness value of the recent ball of 2 mm diameter is $0.0035 \mu m$ [8].

The expected value of the $f_c$ component is proportional to the given mutual diameter differences of balls, as shown in Fig. 4. Additionally, in the case of the same value of mutual diameter differences of balls, a larger number of balls makes the $f_c$ component smaller. However, if the mutual diameter differences of balls are minutely by $0.01 \mu m$, the contributing ratio to reduce the expected value of the $f_c$ component divided by the diminution of the mutual diameter differences of balls in a production lot.

Fig. 8. Relationship between $f_c$ component and diameter difference of balls per one production lot.

4. Measurement of $f_c$ component according to location of balls

4.1. Measurement apparatus for NRRO

Fig. 9 shows an experimental apparatus for investigating the influence of location of balls on the $f_c$ component [2]. As the NRRO measuring device methods for single ball bearing, the following techniques are reported. The first is a measurement technique of radial runout of the spindle axis which is constructed by the rolling bearing to be measured and another rolling bearing [9,10]. The second is an imitative technique of the revolution error measurement method for rolling bearings described in JIS B 1515, using aerostatic Oldham coupling [11]. Using the measurement apparatus in this research, the radial runout of the outer raceway is measured when the inner raceway is rotated. The driving mechanism, location of sensors and data processing method are the same as those of the system which has been developed by one of the authors [12]. By connecting a thin string to the cover of the outer raceway and fixing it to a load cell, the torque of the bearing can be measured at the same time.

The countering forces to the bearing vibration are shear friction of the circumferential air and bending friction of the thin string. So, in comparison with the other techniques, the pure vibration of the bearing can be measured by this structure. Sensors are arranged in two orthogonal directions. The sensor signals are sampled by a rotary encoder pulse.
4.2. Experimental conditions

The main purpose of this research is to confirm the influence of the location of balls on the $f_c$ component. Therefore, the experiment is performed by locating some larger balls (0.1 µm larger in diameter) intentionally. The rolling bearing used here is type No. 695 for a hard-disc-drive spindle motor. Tables 4 and 5 show the experimental conditions and the location of balls respectively. The eight balls in type No. 695 bearing are numbered, and the larger diameter ball is symbolized by “L,” and the normal reference diameter ball, “N” in Table 5.

Lissajous’ plot, determination of the maximum value of the $f_c$ component and fast Fourier transformation (FFT) processing are performed on the computer.

4.3. Experimental results

The results are shown in Fig. 10. The location of balls has an impact on the $f_c$ component although the same number of larger balls are used as shown by the patterns of C and D, E and F and G and H. As a trend, when the larger balls are located serially, the $f_c$ component becomes larger similar to that shown in Fig. 7. This experimental result reveals that the location of balls has a significant effect on the $f_c$ component value even if the mutual diameter differences of balls are the same in a rolling bearing.

5. Conclusions

This paper focuses on the theoretical analysis for the influence of the mutual diameter differences of balls, which cause the retainer revolution run-out component, $f_c$, as the most important factor in NRRO of rolling bearing. Then, the influence of the location of balls is investigated experimentally. The main conclusions are as follows.

(1) The influence of the mutual diameter differences of balls including balls location is clarified statistically. The mutual diameter differences of balls are the most important factor in the case of NRRO.
(2) The $f_c$ component decreases in any case when the number of balls mounted in a rolling bearing increases.
(3) The $f_c$ component decreases when the mutual diameter differences of balls in a production lot are small. However, in an actual number of balls in a rolling bearing, the expected reduction ratio of the $f_c$ component is around 20% of the reduction ratio of the mutual diameter differences of balls.
(4) The influence of location of balls on the $f_c$ component is confirmed experimentally.

References


