

A STUDY ON WN SPIRAL BEVEL GEAR MANUFACTURED BY MACHINING CENTER

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ABSTRACT

Helical gear with arc teeth of point contact is called WN gear derived from initial of inventor Wildhaber and Novikov. Recently, Haseg Gear Co. LTD of Japan has produced WN gear and practically applied for pinion stand of iron making industry. It is reported that this WN gear has higher surface durability than that of involute gears⁽¹⁾. This study is related to the theory of meshing and manufacturing of WN spiral bevel gear with circular arc teeth perpendicular to the teeth trace. WN spiral bevel Gear was actually designed and manufactured by machining center, using a pair of concave and convex end mill with circular axial section. Tooth contact analysis of the above WN spiral bevel gear was carried out. In addition, tooth contact marks were obtained experimentally. Tooth contact marks calculated by TCA agreed well with those obtained experimentally.

Keywords: WN Spiral Bevel Gear, TCA, Machining Center, Circular profile tooth, Circular profile end mill, Tooth Contact Mark

1. INTRODUCTION

Around 1960, Novikov invented a point contact helical gear that has arc teeth in the Soviet Union, and since then effort for practical use of the gear have been promoted all over the world.

Novikov received the Lenin Award for this invention. Since, the US patent was already obtained by E.Wildhaber of Greason Inc. in 1926⁽²⁾, the gear have been called Wildhaber-Novikov gear(hereafter referred as WN gear).

In Japan, Dr Honobe of Hitachi Manufacturing Co. LTD. engaged in development of WN gear and succeeded in practical use of Sinmarc Gear, Haseg Gear Co. LTD. produces the gear at present.

The Sinmarc gear is practically applied, for example, for pinion stand of rolling mill in iron making industries. It is reported that the Gear has higher load capacity than that of involute gear⁽¹⁾.

This study report the theory of meshing and manufacturing of WN spiral bevel gear. The WN Spiral gear is manufactured practically and Tooth Contact Analysis is calculated and the results of TCA agreed well the tooth contact mark obtained by experiment.

Fig.1 is schematic diagram showing Wildhabers idea of WN helical gear. As shown hear, Wildhaber included ides of arc tooth perpendicular to the tooth trace in his application of WN gear. Concave and convex arc tooth perpendicular to the tooth trace do point contact and trace of contact points becomes a line parallel to Pitch line.

Maki M., one of the authors of this report proved correctness of Wildhaber's proposition using vector analysis⁽³⁾.

The proof is as follows: Mathematical definition of the arc tooth perpendicular to tooth trace is that the arc tooth is placed on a plane perpendicular to the tangent of the helix going through center of the arc. Tooth surface of helical gear is composed of helices around the gear axis. The relative velocity of rotations around two axes 1,2 in case of spiral bevel and helical gear consists of a field of circular motion around a straight line, instantaneous axis i.e. ; Pitch line of mating gears. Streamlines of the relative velocity become circles around Pitch line as drawn in Fig.2.

2 . MESHING AND MANUFACTURING THEORY OF WN SPIRAL BEVEL GEAR

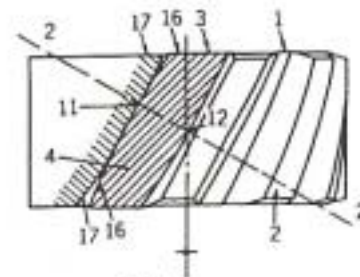


FIG.1

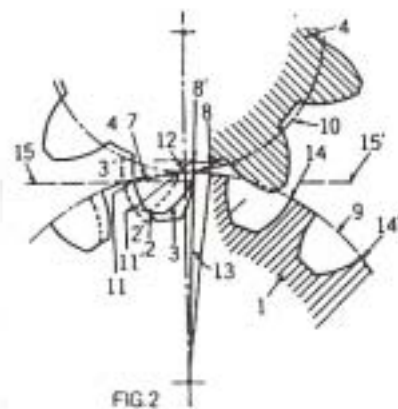


FIG.2

Fig.1 Circular arc tooth perpendicular to the tooth trace drawn in Wildhaber's patent

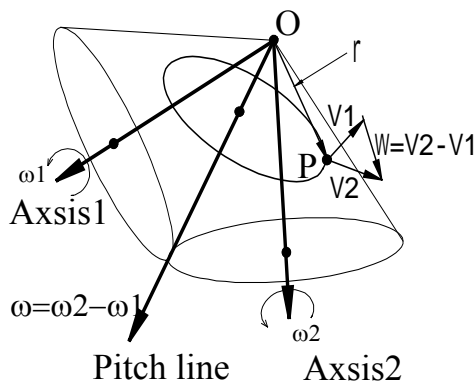


Fig.2. Circular stream line around pitch line of bevel gear and helical gear

This phenomenon is proved by vector equations as follows:
 The velocity of gear 1 and 2 at point P can be written as V_1 and V_2 , and then relative velocity W of gear 1 and 2 at point P is written as,

$$W = V_2 - V_1 \quad \dots (1)$$

Where,

$$V_1 = \omega_1 \times r$$

$$V_2 = \omega_2 \times r$$

$$W = (\omega_2 - \omega_1) \times r$$

Then,

$$W = \omega \times r \quad \dots (2)$$

From equation (2), streamline of the relative velocity becomes circles around Pitch line as drawn in Fig.2.

In case of helical gear, point O of Fig 2 goes far away. Streamline of the relative velocity also becomes circles around Pitch line in case of helical gear.

When the center of arc profile of tooth perpendicular to tooth trace is on Pitch line, any point of the arc of the tooth of spiral bevel gear can satisfy the condition of contact, because that the normal of tooth surface intersects Pitch line. This can be proved as follows;

In Fig. 1, condition of contact at point P is written as follows:

$$n \cdot W = 0,$$

Where,

$$W = \omega \times r$$

$$= \omega \times n$$

Then,

$$n \cdot W = n \cdot (\omega \times n) = \omega \cdot (n \times n) = 0$$

Thus, when the center of arc is on Pitch line, any point of the arc of the tooth of spiral bevel gear can satisfy the condition of contact.

Fig. 3 shows theory of cutting circular arc tooth profile by spherical cutter. In this figure, angular velocity and translation velocity of the relative screw motion around the instantaneous axis are ω and h , respectively. h represents pitch per radian of the relative screw motion and called reduced pitch. Relative velocity of the center of the spherical cutter respect to the work is V_p , and relative velocity of arbitrary point A on the surface of spherical cutter of is V_A . The position vector from the center of the spherical cutter to point A is r . If the unit normal at an arbitrary point A is n , n is expressed as $n = r / r^2$, using the position vector, r .

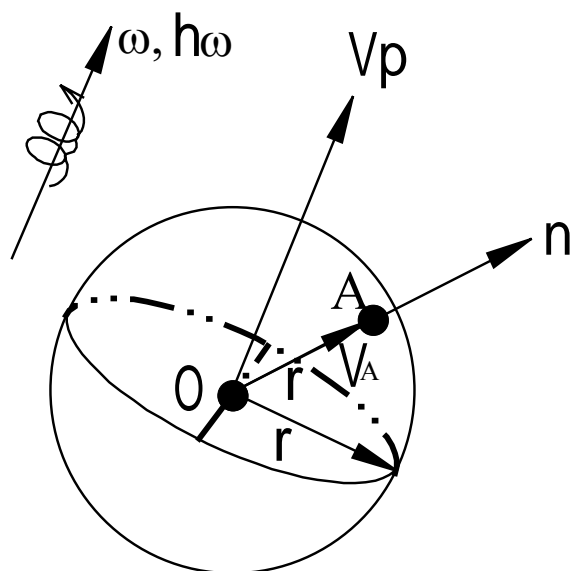


Fig. 3 Cutting Circular arc tooth profile by spherical cutter

The relative velocity of the point A with respect to the work material V_A , is expressed by

$$V_A = V_p + \omega \times r \quad \dots (3)$$

Using relative velocity of the center of the spherical cutter V_p and relative angular velocity around instantaneous axis ω . Condition contact at the arbitrary point A is expressed as follows:

$$n \cdot V_A = 0 \quad \dots (4)$$

Substituting $n = r / r^2$ and equation (3) to equation (4),

$$\left(\frac{r}{\sqrt{r^2}}\right) \cdot (V_p + \omega \times r) = 0 \quad \dots (5)$$

$$\text{Here, } \left(\frac{r}{\sqrt{r^2}}\right) \cdot \omega \times r = \omega \cdot \left\{ r \times \left(\frac{r}{\sqrt{r^2}}\right) \right\} = 0$$

Then, equation (3) is rewritten as

$$\left(\frac{r}{\sqrt{r^2}}\right) \cdot V_p = 0 \quad \dots (6)$$

Equation (6) indicates condition of contact at the arbitrary point A, and represents a plane, which goes through the center of the spherical cutter P, and is perpendicular to the direction of V_p . For this reason, the simultaneous contact line of the spherical cutter surface and surface of the work is of large circle, which is an intersecting line of the plane represented by equation (6) with the surface of spherical cutter.

When spiral bevel gear is processed using a spherical cutter, the contact line of the sphere with the processed spiral bevel gear surface becomes a great circle of the spherical cutter on the plane, which passes through the center of the sphere and is perpendicular to relative velocity V_p between cutter and Spiral bevel gear. This result can be applied to processing WN spiral bevel gear whose tooth profile perpendicular to tooth trace is circular arc.

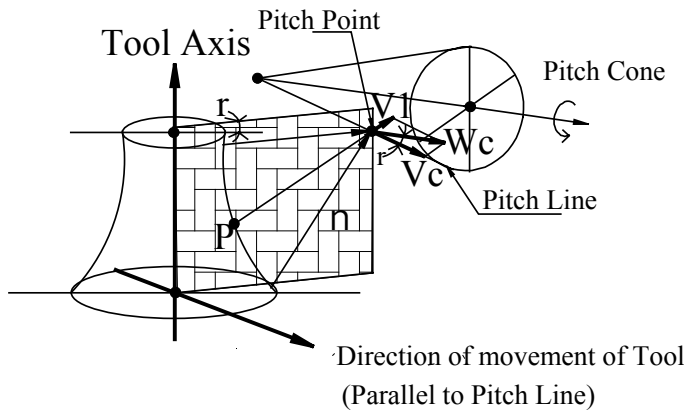


Fig.4 Cutting convex circular arc tooth profile by volcanic shape end mill

Fig.4 shows method of cutting convex circular arc tooth profile of spiral bevel gear by volcanic shape end mill. The volcanic shape end mill has concave circular profile cut by a plane containing tool axis. Suppose that center of concave circular profile is on Pitch point of bevel gear and goes along Pitch line with velocity V_c . The contact line of the volcanic shape end mill with the processed spiral bevel gear surface becomes a circle on the plane, perpendicular to relative velocity W_c between velocity of end mill V_c and velocity of Spiral bevel gear V_1 , from the same consideration as the case of spherical cutter as stated above. From consideration of Fig.2, condition of contact processed circular profile of tooth of spiral bevel gear can fulfill the condition of contact.

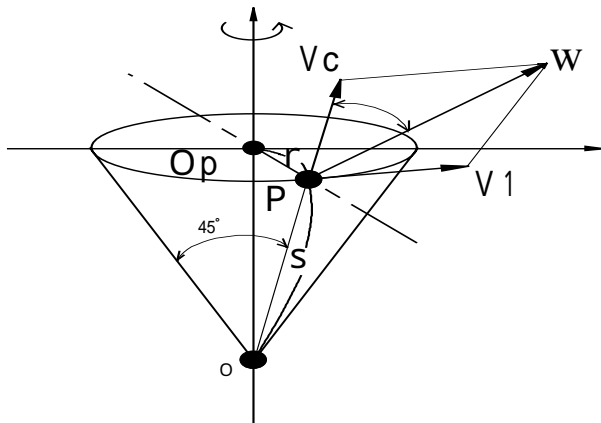


Fig.5 Lead angle of WN spiral bevel gear processed by spherical tool

Fig.5 shows lead angle of WN spiral bevel gear at Pitch point P processed by spherical tool. Supposing that spherical tool goes along Pitch line with velocity V_c and ω_1 is angular velocity of work. r is vector from point O_p to Pitch point P. Absolute value of r is radius of Pitch cone at Pitch point P. Velocity of WN spiral bevel gear at Pitch point P can be written as,

$$|V_1| = \omega_1 \times r \dots \dots (7)$$

Moving velocity of tool along Pitch line V_c can be written as;

$$|V_c| = \frac{ds}{dt} = \frac{ds}{d\theta} \frac{d\theta}{dt} = \frac{ds}{dt} \omega_1 \dots \dots (8)$$

Where S is distance from top point of Pitch cone O to Pitch point P . θ is rotating angle of work of spiral bevel gear, and t

is processing time. Let lead angle at point P be γ ,

$$\tan \gamma = \frac{|V_1|}{|V_c|} = \frac{|r|}{\left| \frac{ds}{dt} \right|} \dots \dots (9)$$

In the case of processing spiral bevel gear by spherical cutter, the contact lines of great circle of spherical cutter change their directions because of changing directions of relative velocity W of center of spherical cutter as explained by Fig.3, because of variation of radius of Pitch circle of Pitch cone of Fig.5.

So then, we can hold circular contact lines of great circle of spherical cutter fixed by controlling velocity of moving of center of spherical cutter by NC machine tool as follows:

From Fig.5,

$$S = \sqrt{2}r,$$

$$\frac{ds}{dt} = \frac{S}{\sqrt{2}} \cot \gamma \dots \dots (10)$$

From equation (10), function $S = S(t)$ has a character of differentiated form having an original function. Function with such characteristics has

the form of e^t .

$$\text{Putting } h = \frac{1}{\sqrt{2}} \cot \gamma,$$

$$S = e^{ht} \dots \dots (11)$$

Then,

$$\frac{ds}{dt} = h e^{ht} = \frac{S}{\sqrt{2}} \cot \gamma \dots \dots (12)$$

As a result of these equations, we can hold circular contact lines fixed by controlling velocity of center of spherical cutter fit to equation(11)

$$S = e^{ht}; \text{ by NC machine tool.}$$

In the case of practical processing, the variable t should be exchange for rotating angle of work by equation $\theta = \omega_1 \times t$, where ω_1 is arbitrary constant angular velocity of work of spiral bevel gear.

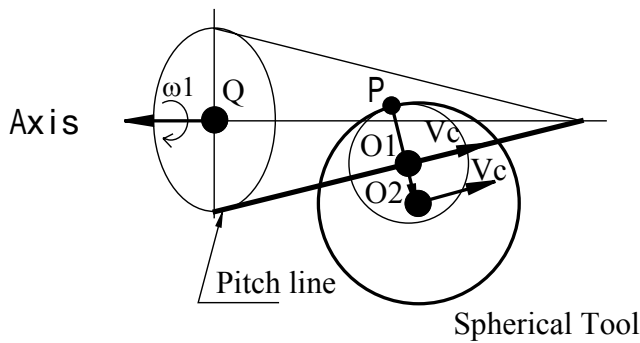


Fig.6 Processing method of mating teeth of WN spiral bevel gear

For this spiral bevel gear is called WN spiral bevel gear, trace of mating contact points of circular arc teeth should be a line parallel to Pitch line.

This proposition can be fulfilled as follows:

At first, connecting line of center of spherical cutter O2 and center of circular axial profile of volcanic shape end mill O1 intersect Pitch line as shown in Fig.6.

Spherical cutter should move parallel to Pitch line, with the same velocity of volcanic shape end mill at point O1. The center of spherical cutter should be in position that the sphere of cutter surface comes in contact with circular arc of convex tooth of mating gear at point P. Three points O2, O1 and P lies on a line, which is center line of spherical cutter.

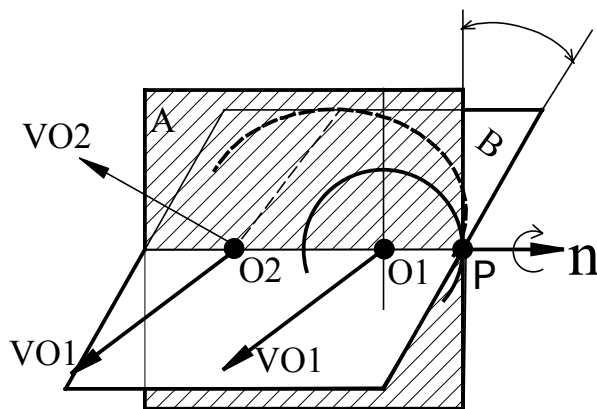


Fig.7 Convex and concave circular profile of mating tooth

When spherical cutter moves so that point O1 on center line of spherical cutter has the same velocity of volcanic shape end mill VO1, velocity of center of spherical cutter O2 ;VO2 changes slightly from VO1, because of distance O1 and O2.

Let distant vector O1 to O2 be r , VO2 is written as:

$$VO2 = VO1 + \omega_1 \times r \dots (13)$$

As vector r lies on common normal n of mating circular teeth profile, VO2 is perpendicular to common normal n . So, the plane on which circular profile of concave tooth rotates around normal n by small angle as shown in Fig.7.

Because that common normal n intersects Pitch line as stated above concerning Fig.6, contacting point of mating convex and concave circular profile can fulfill the condition of contact as explained as concerning with Fig.2.

By the theory stated in this chapter, we can process WN spiral bevel gear, whose profile of mating teeth perpendicular to tooth trace has concave and convex circle.

The circles contact with each other on the point P, which moves along a straight line parallel to Pitch line.

3. PROCESSING OF TOOLS AND SPIRAL BEVEL GEAR

The dimensions of WN spiral bevel gear are as follows:

Normal pressure angle ; $= 30^\circ$

Helix angle on Pitch line ; 30°

Number of teeth; Z1 and Z2 40.



Fig.8 Completed volcanic shape end mill and spherical end mill

Tools for spiral bevel gear was processed by wire cut E.D.M, Sodic, Co.LTD Japan. Diameter of Volcanic shape end mill is 12 mm. Spherical end mill is made from double blade, high slotting end mill whose diameter is 16 mm.



Fig.9 Processing of WN spiral bevel gear by marching center

Fig.9 shows the processing of WN spiral bevel gear by marching center. Machine tool is Makino milling cutter (MSA40), Japan. Control unit is NIKKEN industries Japan and work material is carbon steel S25C. Cutting conditions are as follows: Cutting speed: 25 m/min, feed: 0.05 mm/tooth. Simplified wet process (machine oil)

4. TOOTH CONTACT ANALYSIS

Maki M., one of the authors of this study is the first reporter of Tooth Contact Analysis, (TCA) in Japan⁽⁴⁾. The authors calculated how the tooth

contact mark changes according to the displacement of the axes of the gearing. The contact mark calculated is illustrated according to V-H test of Glaeson works⁽⁵⁾. Fig.13 shows the results of TCA calculated by the authors.

The authors manufactured the apparatus for experiment of tooth contact analysis as shown in Fig.14, and tooth contact marks of WN spiral bevel gear are obtained. Red lead was painted on tooth surface of gear, and mating teeth were pressed with each other. The contact marks were subscribed to cellophane tape. The contact marks coincide well with the results of calculation of TCA.

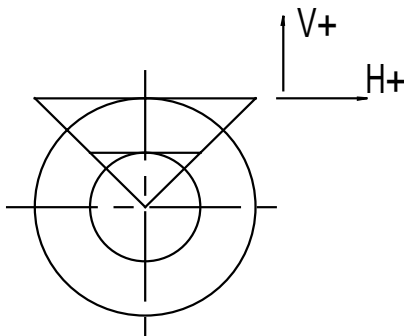


Fig.10 Expression of displacement of gear to gear

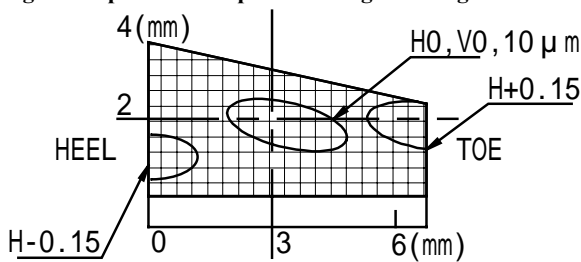


Fig.11 Result of TCA of WN spiral bevel gear



Fig.12 Apparatus for experiment of TCA applying Milling machine

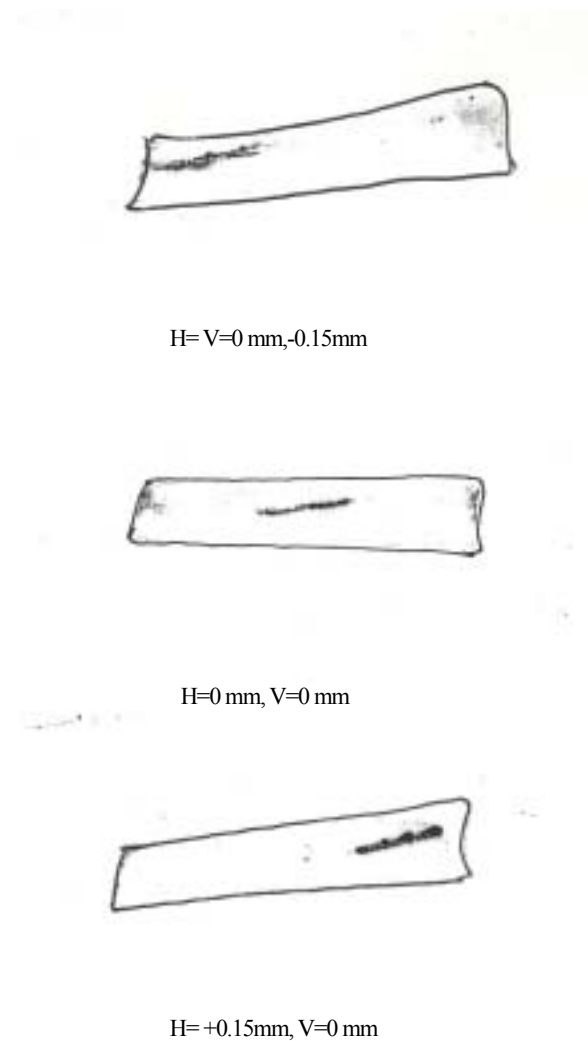


Fig.13 Tooth contact marks of WN spiral bevel gear

5. CONCLUSIONS

Based on the idea of patent application of Wildhaber and a theory developed by Maki, study on WN spiral bevel gear with arc teeth perpendicular to the teeth was performed and the following conclusions were obtained.

- (1) When spiral bevel gear is processed using a spherical cutter, the profile of tooth perpendicular to tooth trace becomes a great circle of the spherical cutter on the plane, which is perpendicular to relative velocity V_p between cutter and Spiral bevel gear passing through the center of the sphere.
- (2) When spiral bevel gear is processed using a volcanic shape end mill having the concave circular profile cut by a plane containing tool axis, the profile of tooth perpendicular to tooth trace becomes the concave circular profile end mill cut by a plane containing tool axis.
- (3) When the center of arc profile of tooth perpendicular to tooth trace is on Pitch line, any point of the arc of the tooth of spiral bevel gear can satisfy the condition of contact, because that the normal of tooth surface intersects Pitch line.
- (4) We can hold circular contact lines fixed by controlling velocity of

center of spherical cutter fit to equation $S = e^{ht}$; by NC machine tool.

(5) Using the developed processing system based on the theory stated above, WN spiral bevel gear with arc teeth perpendicular to tooth trace were actually designed and manufactured.

(6) Tooth contact analysis of the WN spiral bevel gear was carried out. In addition, a tooth contact tester was manufactured to compare the results of tooth contact of the gears. The results obtained experimentally were in good agreement with those obtained analytically.

5. REFERENCES

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